Queen's Hill Primary School and Nursery Calculation and Number Policy

2019-2020

The overall aim is that by the end of Year 6 all children will:

- have a secure knowledge of number facts and a good understanding of the four operations;
- be able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- be able to use a calculator effectively when required, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

The CPA approach

One of the key learning principles in effective mathematics teaching is the concrete pictorial abstract approach, often referred to as the CPA approach. This is integral throughout a child's learning journey, from EYFS to Year 6.

The concrete-pictorial-abstract approach, based on research by psychologist Jerome Bruner, suggests that there are three steps (or representations) necessary for pupils to develop understanding of a concept. Reinforcement is achieved by going back and forth between these representations. Within the mastery approach to teaching mathematics, it is commonly known that presenting them together has the most positive impact on a child's learning.

Concrete representation

The active stage - a student is first introduced to an idea or a skill by acting it out with real objects. In division, for example, this might be done by separating apples into groups of red ones and green ones or by sharing 12 biscuits amongst 6 children. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

Pictorial representation

The iconic stage - a student has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem. In the case of a division exercise this could be the action of circling objects.

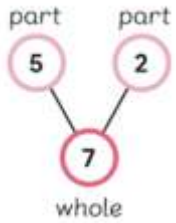
Abstract representation

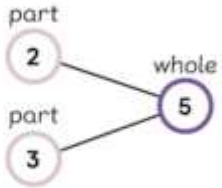
The symbolic stage - a student is now capable of representing problems by using mathematical notation, for example: 12 ÷ 2 = 6 this is the ultimate mode, for it is clearly the most mysterious of the three.

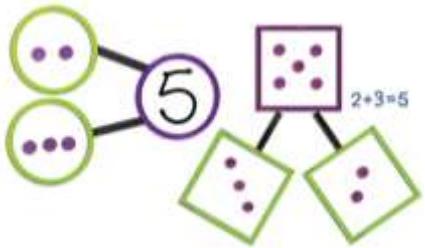
Number Bonds

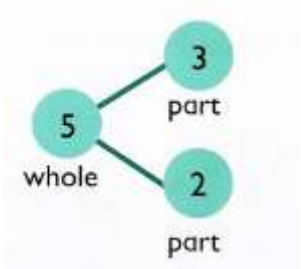
Number bonds refer to how numbers can be combined or added up. It is the 'part-part-whole' relationship of numbers. When talking about number bonds in maths mastery we are referring to how numbers join together and how they can be split up. A lot of emphasis is put into number bonds from EYFS so that children can build up their number sense prior to learning addition and subtraction. In the early stages students would be introduced to number bonds with concrete experiences, for example children could be given 6 linking cubes and guided to understand that 2 and 4 make 6, but that 1 and 5 also make 6.

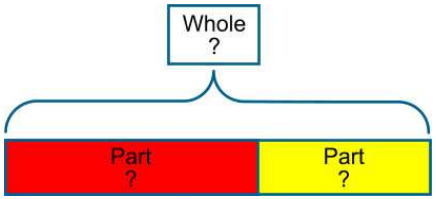
The part-part-whole model can be shown in the following ways:



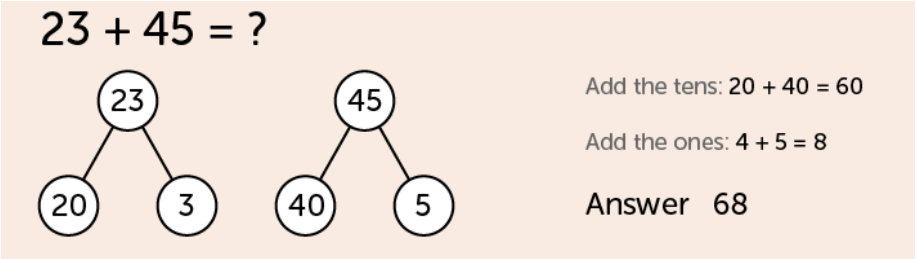








The mastery of number bonds is an important foundation that must be continually revisited throughout the child's school journey. It is required in subsequent mathematical learning and as a basis in the development of mental strategies. For example, children need to know that 7 + 3 = 10 to know that 7/10 + 3/10 = 10/10= 1 whole. A strong number sense allows students to decide what action to take when trying to solve problems in their head.



Year group number bond areas of focus:

Year Group	Teaching focus	Consolidation focus
EYFS	Finding one more and one less with numbers to 10	
Year 1	Number bonds to 10	Finding one more and one less with numbers to 10
Year 2	Number bonds to 20	Number bonds to 10
Year 3	Number bonds to 100 and 1000 Number bonds to 12 and 60 (to help with reading time) Number bonds to 1 (e.g. 1 tenth + 9 tenths makes one whole)	Number bonds to multiples of 10
Year 4	Number bonds to 100, 1000 and 10,000 Number bonds with decimals and fractions to whole numbers (any given whole number) Number bonds to 12 and 60 (to help with reading time)	Number bonds to 100 and 1000 Number bonds to 1 (e.g. 1 tenth + 9 tenths makes one whole)
Year 5	Number bonds with percentages	Number bonds to 100, 1000 and 10,000 Number bonds with fractions to whole numbers (any given whole number)
Year 6	All number bonds (e.g. multiples of 10, 100 and 1000) Number bonds to a whole Number bonds	All of the Year 3, 4 and 5 teaching focus objectives

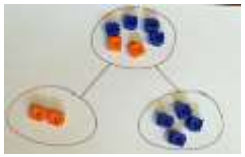
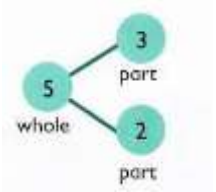
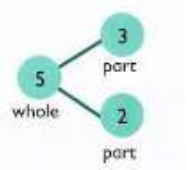
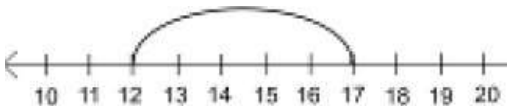
Times Tables

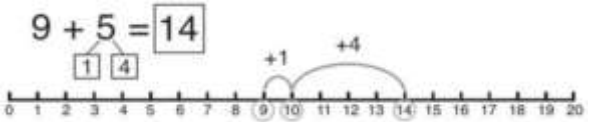
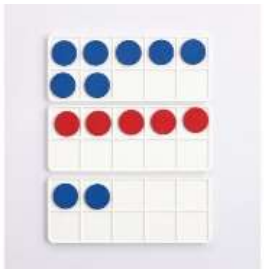
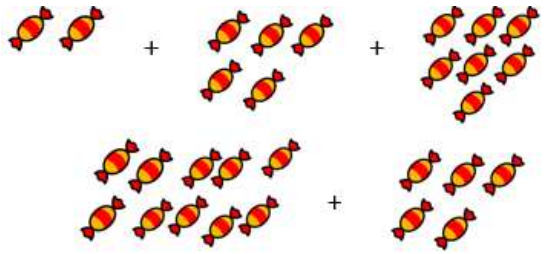
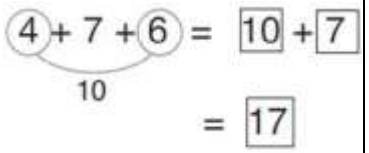
It is essential for all pupils to have a firm grasp of all times tables by the time the reach the end of Year 4. Not only is this important for their multiplication work, but is crucial when exploring other topics later in their school time, such as ratio and proportion, fractions, algebra, scale factor and much more!

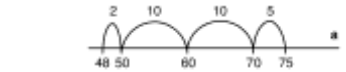
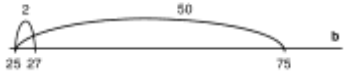
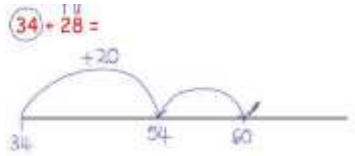
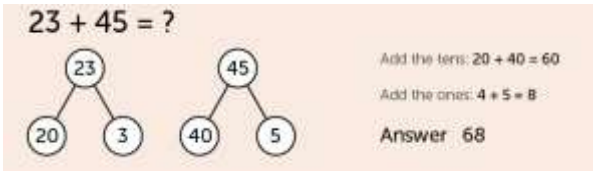
Following our whole-school focus, teachers ensure that pupils have daily opportunities to practice their times tables. These are the following year group times table expectations:

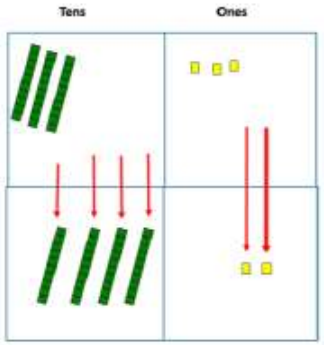
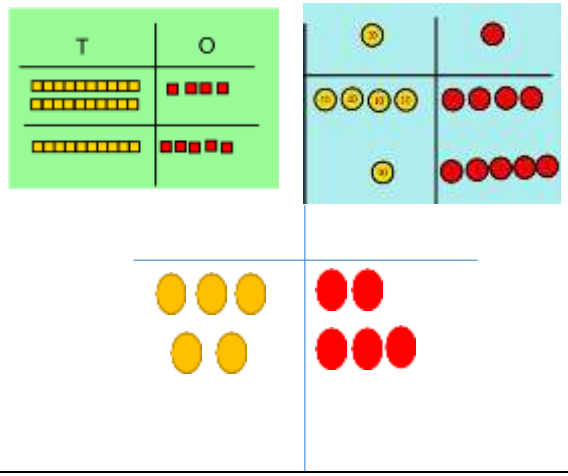
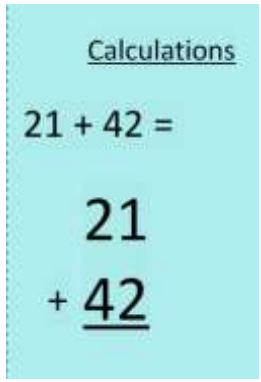
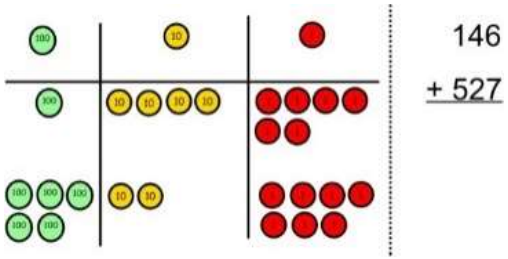
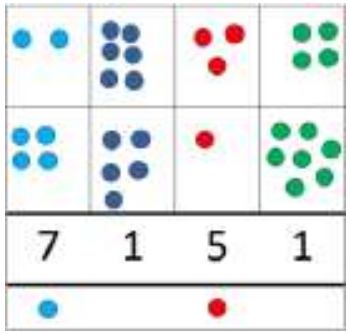
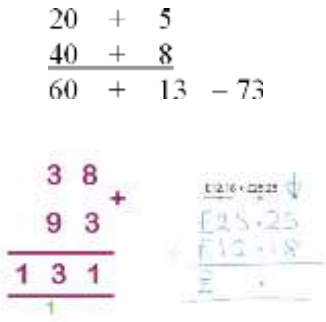
Year group	Teaching Focus	Consolidation
EYFS	Counting forwards and backwards in ones (including zero)	Being able to place numbers in order and count objects with one to one correspondence accurately
Year 1	Counting in 2s, 5s and 10s	Counting forwards and backwards in 1s (to include some negative numbers)
Year 2	Counting in 2s, 5s and 10s Counting in 50s and 100s	Counting forwards and backwards in 1s (to include some negative numbers)
Year 3	Counting in 3s, 4s and 8s Counting in multiples of 25 and any multiple of 10 (e.g. 30, 60, 90…)	Counting in 1s, 2s, 5s and 10s Counting in 50s and 100s
Year 4	Counting in 6s, 7s, 9s, 11s and 12s	Counting in 1s, 2s, 3s, 4s, 5s and 10s Counting in multiples of 25 and any multiple of 10 (e.g. 30, 60, 90…)
Years 5 and 6	Counting in any given multiples	Recall of taught times table facts

Addition

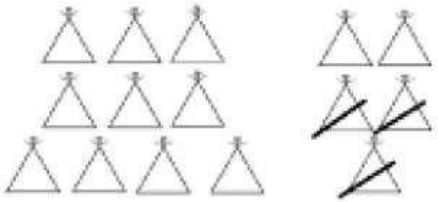
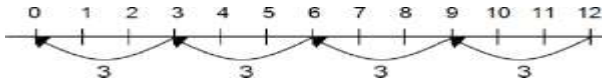
Step	Strategies	Concrete	Pictorial	Abstract
1	Combining two amounts (both augmentation in which two groups are combined and aggregation in which one group is added)	Use cubes or other objects to add two numbers together as a group or in a bar.   	Use pictures to add two numbers together as a group or as a bar.  	Use the part-part whole diagram as shown above to move into the abstract.  $4 + 3 = 7$ $10 = 6 + 4$
2	Starting at the bigger number and counting on (number line strategy)	Children use number lines and practical resources to count on from a bigger number. The teacher models the use of the number line as they do this.  	Start at the bigger number on the number line and count on in ones or in one jump to find the answer. Children use own number line to count on. 	Children can identify that they need to identify the bigger number and count on from that (both mentally and/or on a number line). $5 + 12 = 17$

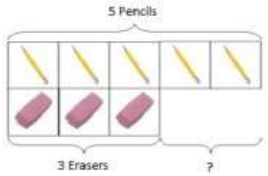
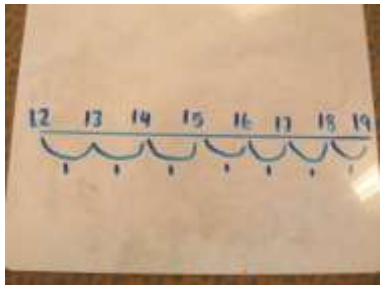
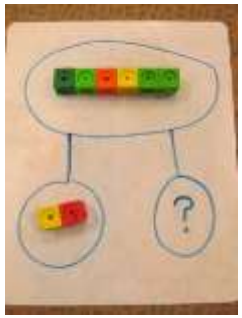
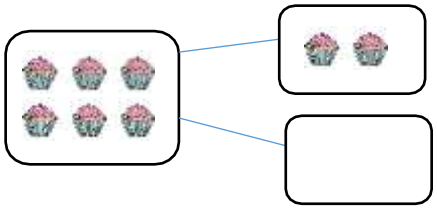
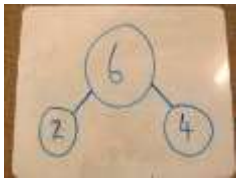
3	Regrouping to make ten	Find the biggest number. Count on to make ten and then see what is remaining.  	Use pictures of a number line. Regroup of partition the smaller number to make 10 (bridge 10) 	If I am at seven, how many more do I need to make 10. How many more do I add on now? 7 + 4 = 11
4	Adding three single digits	Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.  	Add together 3 groups of objects. Draw a picture to recombine the groups to make 10. 	Combine the two numbers that make 10 and then add on the remainder. 

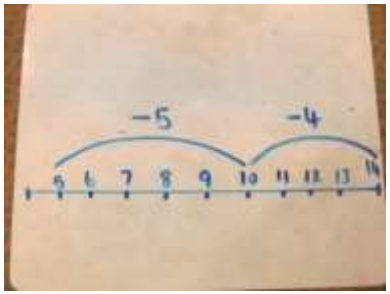
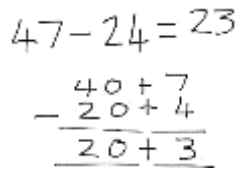
5	The empty number line	Start with the bigger number. Jump up to the nearest multiple of 10, then add the remaining amount.   	When children are confident 'making 10', steps in addition can be  recorded on a number line to improve efficiency. Encourage children to apply their knowledge of making 10 so they bridge 10.	Children are able to identify that they need to start with the biggest number, regardless of the way that the sentence is written. 28 + 34 = 62 34 + 28 = 62 62 = 28 + 34
6	Partitioning	Child learn that they can partition a number has 2 digits or more to another number by partitioning. They should learn to add the smallest value first. 	Children partition numbers before adding them together  	43 + 14 = 57 3+4 = 7 40+10 = 50 50+7 = 57

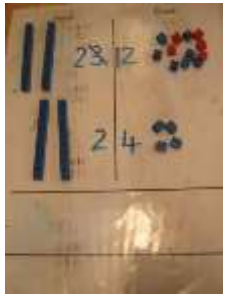
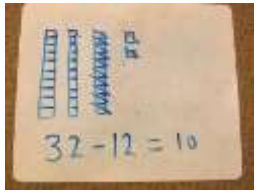
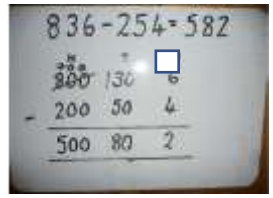
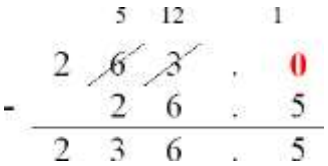
7	Column method (no renaming)	Add together the ones first then add the tens using Base 10 and place value counters. 	Children are able to draw pictorial representations of place value counters and Base 10, understanding the ones are added first. 	Children can add two digits together, ensuring they start with the ones. 
8	Column method (with renaming)	In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'rename 10 ones as 1 ten' or 'rename 10 tens, 1 one hundred'. 	Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding. 	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 

Subtraction

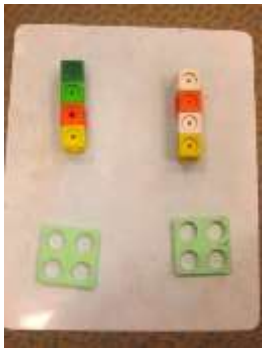
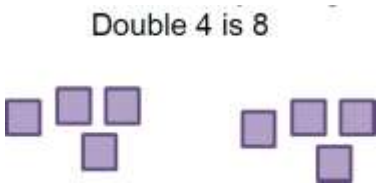
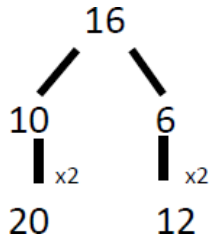
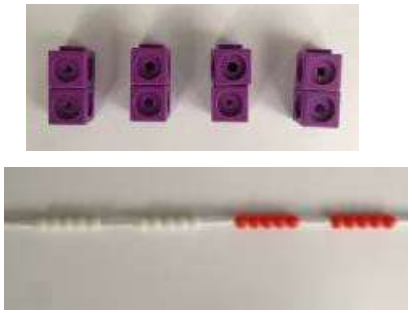
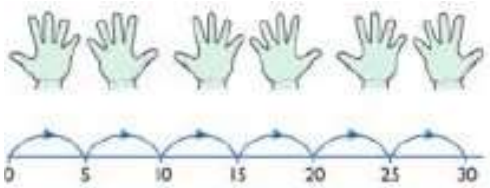
Step	Strategies	Concrete	Pictorial	Abstract
1	Taking away ones	Use physical counters, objects, cubes etc. to take away ones 	Cross out drawn objects to show what has been taken away. 	18 -3= 15 8 – 2 = 6
2	Counting back	Make the larger number in your subtraction . *With counters*, move away from the group the amount that you take away.  *With a bead string,* move the beads along the string as you count backwards 	Count back on a number line or number track. Start at the larger number and count back to the smaller number showing the jumps on the number line. This can progress all the way to counting back using two 2 digit numbers. 	Put 13 in your head, count back 4. What number are you at? Use your fingers to help. 13 – 4 =

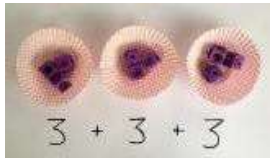
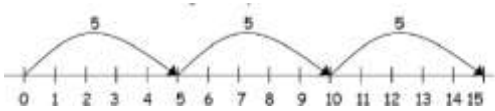
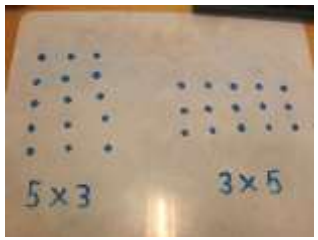
3	Find the difference	Use objects to compare amounts by finding the difference.  	As children have represented two amounts using a concrete material or bar model, they need to learn to **count on** from the smaller amount to the greater amount. 	Hannah has 19 grapes and Helen has 12 grapes. How many more grapes does Hannah have than Helen? Find the difference. 19 – 12 = 7
4	Part-part-whole model	Link to addition - use the part whole model to help explain the inverse between addition and subtraction. If 6 is the whole and 2 is one of the parts. What is the other part? 	Use a pictorial representation of the objects within the model. 	Tom had a party. He made 6 cupcakes for his party. At the end of the party there were two left. How many were eaten? 6 – 2 = 4 

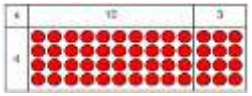
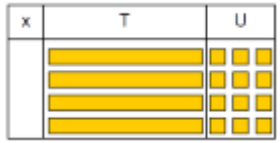
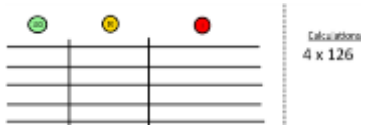
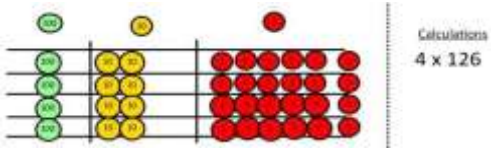
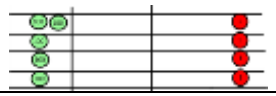
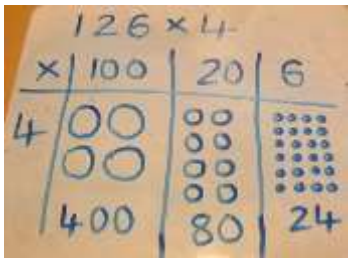
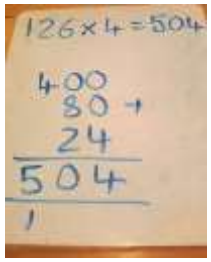
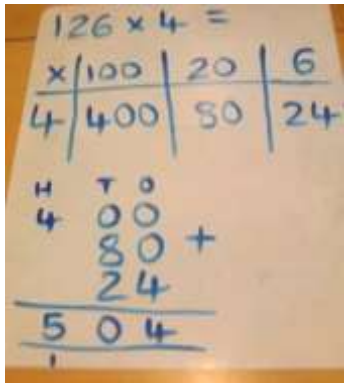
5	Make ten	Children start by making 14 on the tens frame. They take away 4 to make 10 and then they take away the remaining 5. 	As with the tens frames, children should be able to follow this method on the number line. 	14 – 9 = ? How many do we take off to reach the next 10? How many do we have left to take off?
6	Column method (no renaming)	Children should use Base 10 and a Base 10 mat. They should make the bigger number. Show how to partition numbers to subtract. 	Children should also draw Base 10 or place value counters alongside their written methods to help to show their working.  > 	Most children will go straight to the short method. For some children, they may need to start with the expanded method (e.g. some children from Y3/4).  

8	Column method (renaming)	Children should use Base 10 and a Base 10 mat. Children should make the greater value on the mat. They should start by subtracting the ones, then the tens and so on. If they need to 'rename', children must swap their counters, e.g. swap 1 ten for 10 ones and place these in the ones column.  >  > 	Children should draw Base 10 and a Base 10 mat. Children can cross out the sticks as they are used.  When confident, children can find their own way of recording their jottings or ideas.	Some children may begin by using the expanded method, but the majority with begin with the short method.  Short method:  This also applies for decimals: 

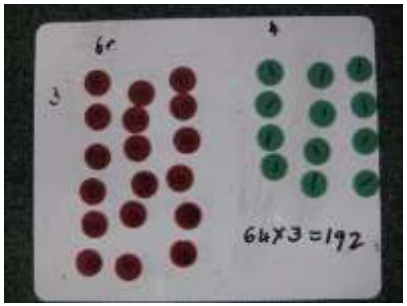
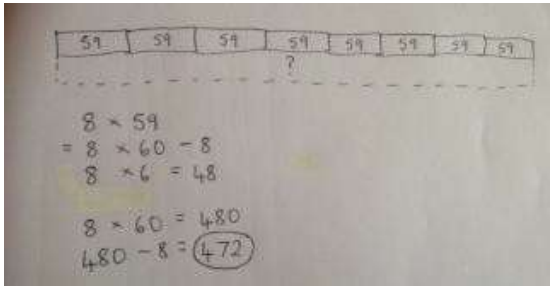
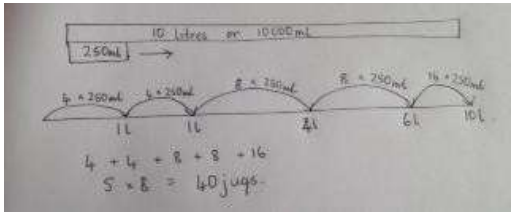
Multiplication

Step	Strategies	Concrete	Pictorial	Abstract
1	Doubling	Use practical strategies to show how to double a number 	Draw pictures to show how to double numbers. 	For numbers greater than 10, partition the number and then double each part before combining it back together. 
2	Counting in multiples	Count in multiples supported by concrete objects in equal groups. 	Use a number line or pictures to continue support in counting in multiples. It is good practice to show the concrete objects above the number line (as in the image below). 	Count in multiples of a number aloud and write sequences with multiples of numbers. Missing number sequences should also be used. 5, 10, 15, 20, 25 5, __, 15, 20, ___

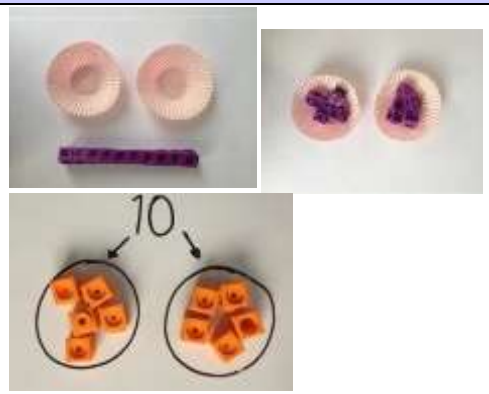
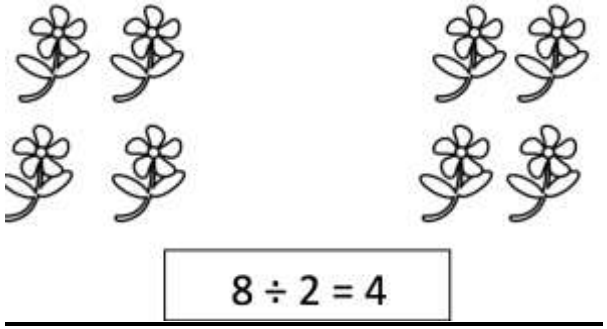
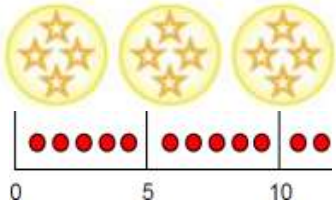
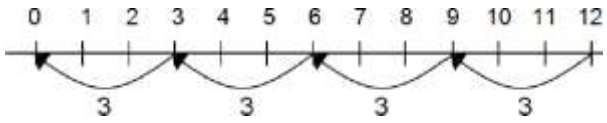
3	Repeated addition	Use different objects to add equal groups together.   	Use a number line or pictures to continue support in counting in multiples. It is good practice to also show the concrete above the number line.  	Write addition sentences to describe objects and pictures. 
4	Arrays (showing commutativity)	Create arrays using counters and cubes to show multiplication sentences.  	Draw arrays (in **different rotations**) to find commutative multiplication sentences. A link should be made between arrays and the area of rectangles.  	Use an array to write multiplication sentences and reinforce repeated addition. 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3= 15 5 x 3 = 15 3 x 5 = 15 Children should also be shown these sentences written in reverse: 15 = 5 + 5 + 5

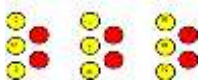
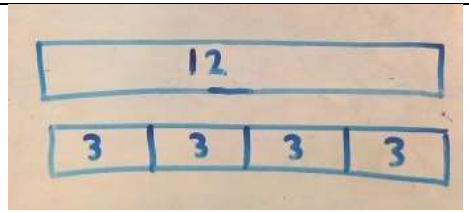
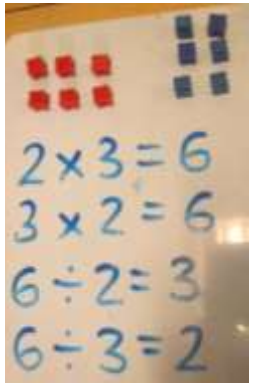
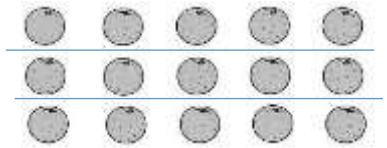
5	Grid method	Link the grid method to arrays initially.  *10 rows of 4 and 3 rows of 4.*  Next, use Base 10 for a more compact method. *4 rows of 13.* Then move to using place value counters to show how to find groups of a number.  *First we need 4 rows because we are multiplying by 4.*  *Fill each row with 126. Add up each row starting with the ones, making any exchanges needed. Then you have your answer.* 	Children can draw their method using place value counters. They can use colours to show the different place value counters or they can just draw circles in columns to show their thinking.  	Children can now show their calculation using the grid method. Children must always start by multiplying the ones. *For multiplying by a single digit number:* **210 + 35 = 245**  *For multiplying by a two or three digit number:*

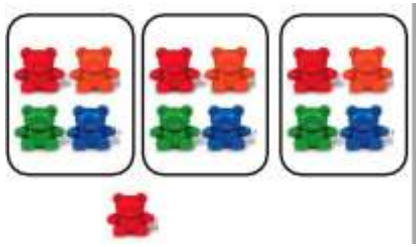
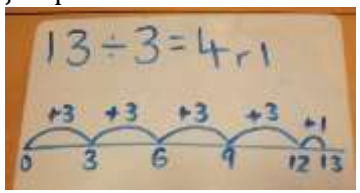
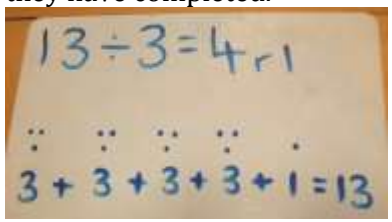
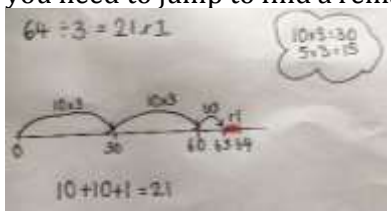
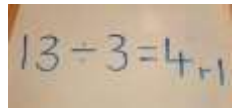
X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

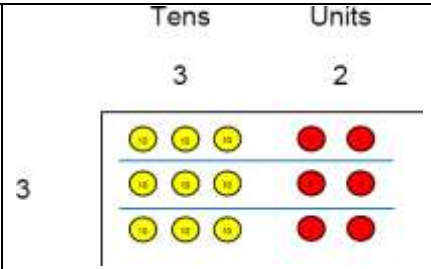
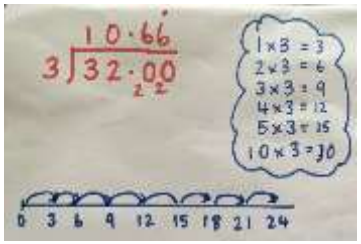
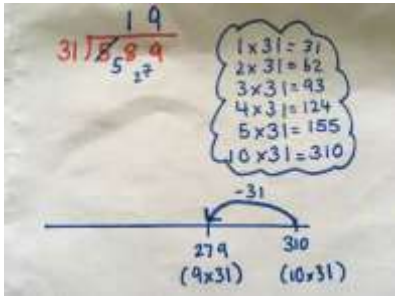
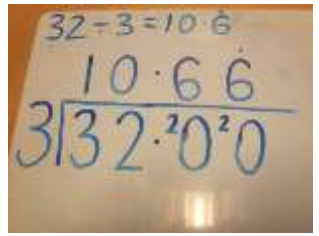
6	Column multiplication	Children can continue to be supported by place value counters. It is essential they start by multiplying the ones. 	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  	Start with the expanded column method. Some learners may feel comfortable using this in the long-term. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768 Then children move to the compact method: 502 x 336 3012 15060 + 150600 168672

Division

Step	Strategies	Concrete	Pictorial	Abstract
1	Sharing objects into groups (to include halving)			Share 8 flowers between two vases. 8 ÷ 2 = 4
2	Division by grouping (including on number line, leading to chunking in KS2)	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  With larger numbers:	Use a number line to show equal groups  Show equal groups using a bar model:	Divide 12 into 3 equal groups 12 ÷ 3 = 4 How many are in each group?

		96 ÷ 3 = 32 		
3	Division within arrays	Link multiplication to division. Look at arrays and explore the different number sentences that can be written (both multiplication and division) 	Children draw arrays and use lines to separate the amount into groups to find both multiplication and division sentences : 	Find the inverse and link multiplication and division: 5 x 3 = 15 3 x 5 = 15 15 ÷ 5 = 3 15 ÷ 3 = 5 Although there are four mathematical sentences, children should also be taught they can be written as follows: 15 = 5 x 3 15 = 3 x 5 3 = 15 ÷ 5 5 = 15 ÷ 3

5	Division with a remainder (leading onto chunking)	13 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots to represent the calculation that they have completed.  Jump forward in chunks (groups of multiples) on a number line and then see how many more you need to jump to find a remainder.  	13 ÷ 3 = Complete written divisions and use the shorthand 'r' for remainder 13 ÷ 3 = 4 r 1 ↑ ↑ ↑ ↑ dividend divisor quotient remainder 

6	Short division		Some children may find it beneficial to note down key multiples of the number that they are dividing by in a 'thought bubble'.  The use of a 'thought bubble' becomes all the more important when children are dividing by a number which isn't a well-known times table to them. From finding this number multiplied by 1, 2, 3, 4, 5 and 10, they can then manipulate these numbers to easily find other multiples on a number line and spot patterns. 	Written bus stop method 

OPTIONAL long division guidance **(Years 5/6 only)**

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)2 7 8	h t o 1 2)2 7 8 - 2 0	h t o 1 8 2)2 7 8 - 2 ↓ 0 7
Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.

Divide.	Multiply & subtract.	Drop down the next digit.
h t o 1 3 2)2 7 8 - 2 0 7	h t o 1 3 2)2 7 8 - 2 0 7 - 6 1	h t o 1 3 2)2 7 8 - 2 0 7 - 6 1 8
Divide 2 into 7. Place 3 into the quotient.	Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the 1 leftover ten.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8	h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8 - 1 8 0	h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	There are no more digits to drop down. The quotient is 139.