EYFS – Year 6

Queen's Hill Primary and Nursery School
2014-15

Calculation Policy

The overall aim is that by the end of Year 6 all children will:

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- use a calculator effectively when required, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

Calculations

Addition

Stage 1: Concrete

Children use real life objects and apparatus to explore the different models of addition.

1. Augmentation in which two groups are combined:

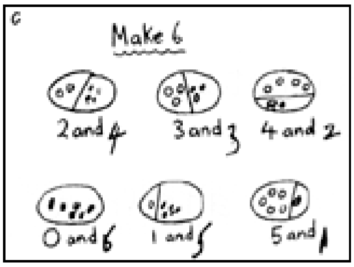
There are 3 footballs in the red basket 2 footballs in the blue basket. How many footballs are there altogether?

2. Aggregation in which one group is added to:

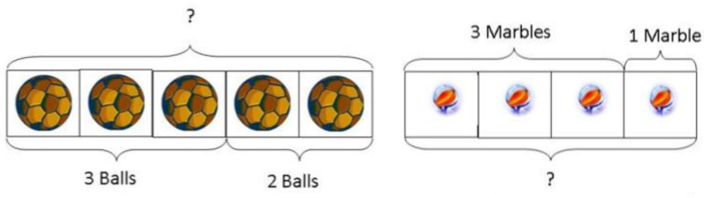
Peter has 3 marbles. Harry gives Peter 1 more marble. How many marbles does Peter have now?

Stage 2: Pictures

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.



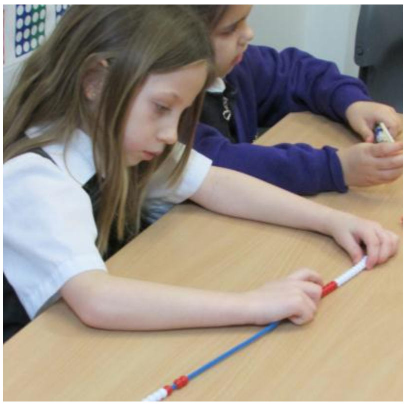
Singapore Bar Model



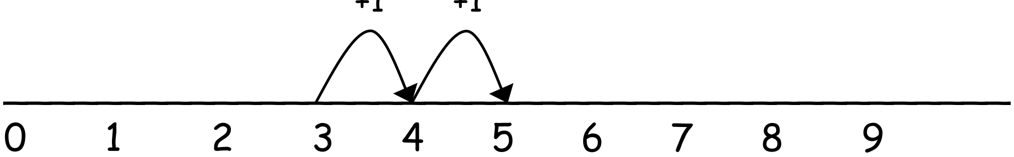
Stage 3: The number line

Children use number lines and practical resources to support calculation and teachers *demonstrate* the use of the number line.

Using bead string as a number line.

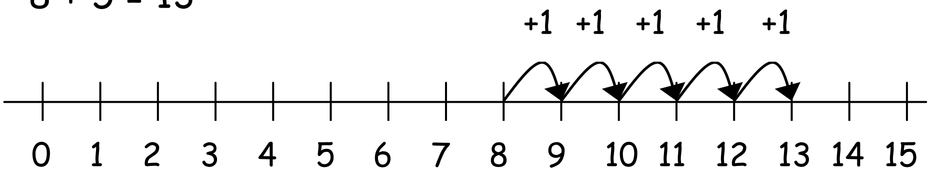


3 + 2 = 5



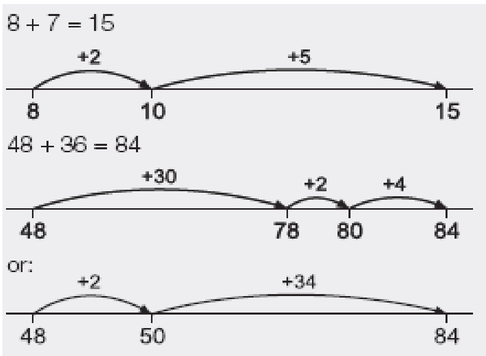
Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

8 + 5 = 13



Stage 4: The empty number line

Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.



Stage 5: Partitioning

Children should be able to partition 2-digit/3-digit numbers before adding them together.

Some children find it useful to record partitioning in this way:

47

40 7

Leading to this ***optional*** method for recording addition if children are not yet ready for the next step:

47 + 76 =

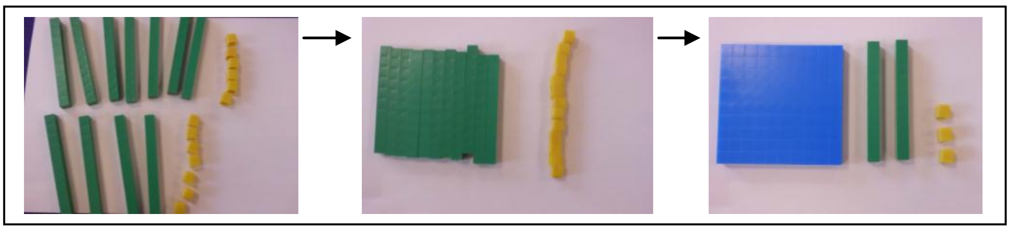
40 + 70 = 110 7 + 6 = 13

110 + 13 = 123

However, it is expected that all children will progress to recording the steps for addition as laid out below.

Add the ones and then the tens to form partial sums and then add these partial sums.

47 + 76 =
7 + 6 = 13
40 + 70 = <u>110</u>
123



Stage 6: Expanded method in columns

Write the numbers in columns, adding the ones first:

```
  TU
  67
+ 24
  11  ( 7 +  4) Ones (Units)
  80  (60 + 20)  Tens
  91
```

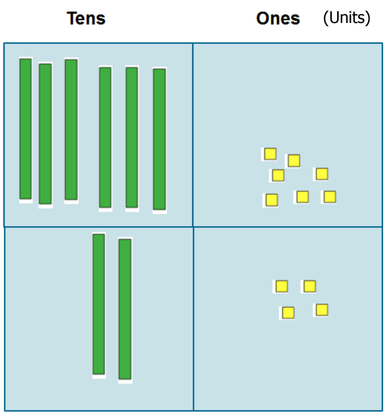
```
  HTU
  267
+  85
   12  ( 7 + 5) Ones (Units)
  140  (60 + 80) Tens
  200            Hundreds
  352
```

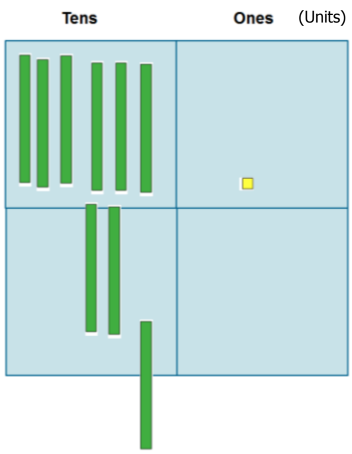
Stage 7: Column method

In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.

```
  TU
  67
+ 24
```

Model with Base 10 apparatus first:





Exchange ten 1s for a 10.

```
  TU
  67
+ 24
  91
   1
```

Children should extend the carrying method with any number of digits.

```
  HTU
  783
+  42
  825
   1
```

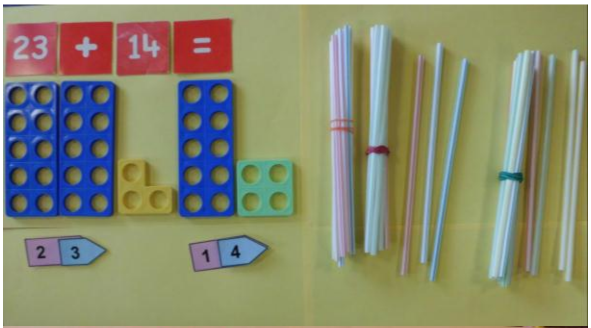
```
  HTU
  367
+  85
  452
  1 1
```

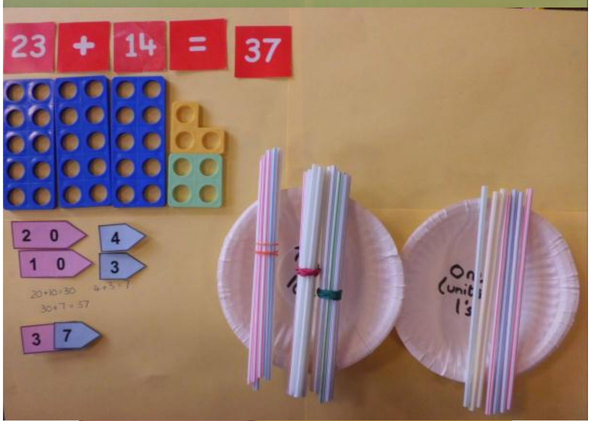
$$\begin{array}{r} \textbf{ThHTU} \\ 7648 \\ +\ \ 1486 \\ \hline 9134 \\ \hline {\scriptstyle 1\ 1\ 1} \end{array} \qquad \begin{array}{r} \textbf{ThHTU} \\ 6584 \\ +\ \ 5848 \\ \hline 12432 \\ \hline {\scriptstyle 1\ 1\ 1} \end{array} \qquad \begin{array}{r} \textbf{ThHTU} \\ 42 \\ 6432 \\ 786 \\ 3 \\ +\ \ 4681 \\ \hline 11944 \\ \hline {\scriptstyle 1\ 2\ 1} \end{array}$$

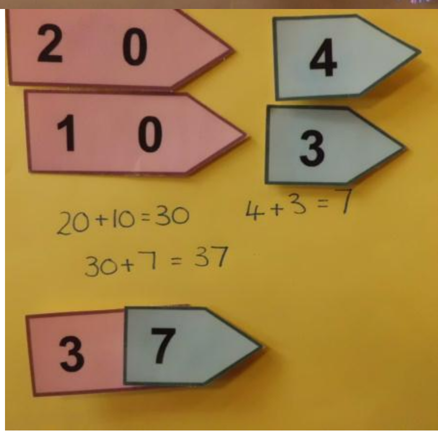
Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

Models and Images

It is important for children to have experienced a range of models and images to ensure that they develop fluency and a secure understanding of addition and place value.





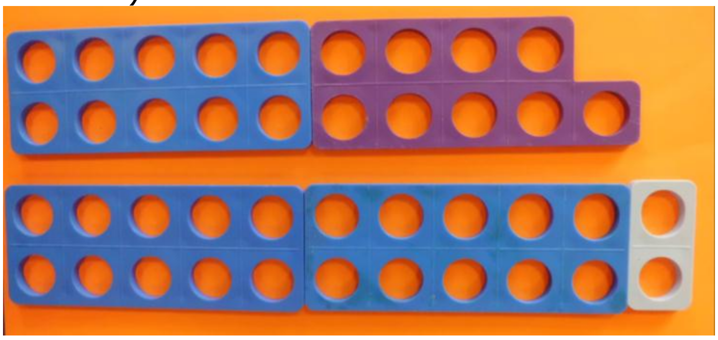


<u>Subtraction</u>

Stage 1: Concrete

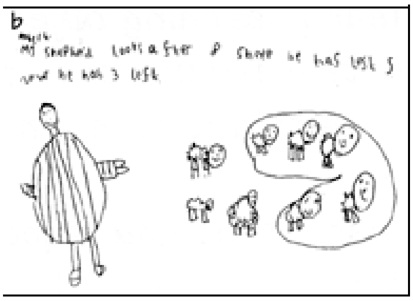
Children use real life objects and apparatus to explore the different models of subtraction.
Removing items from a set (reduction or taking away):

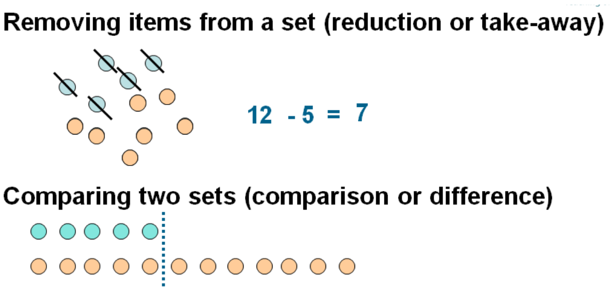
Comparing two sets (comparison or difference):



Stage 2: Pictures

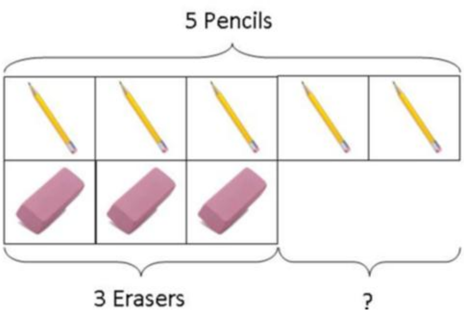
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.





Singapore Bar Model:

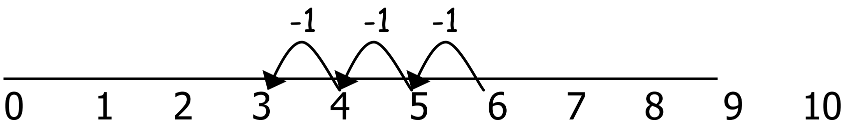
- Peter has 5 pencils and 3 erasers. How many more pencils than erasers does he have?



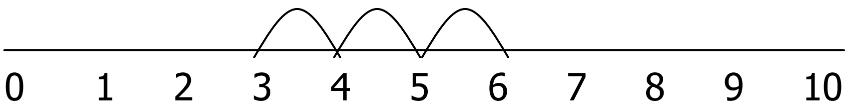
Stage 3: The number line

Children use number lines and practical resources to support calculation and teachers *demonstrate* the use of the number line.

6 – 3 = 3

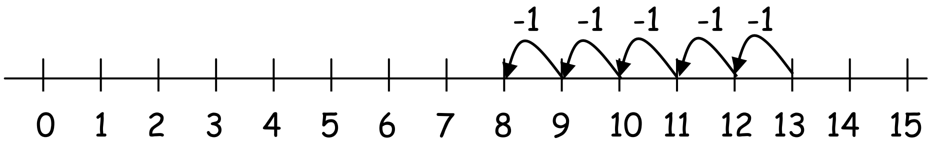


The number line should also be used to show that 6 - 3 means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.



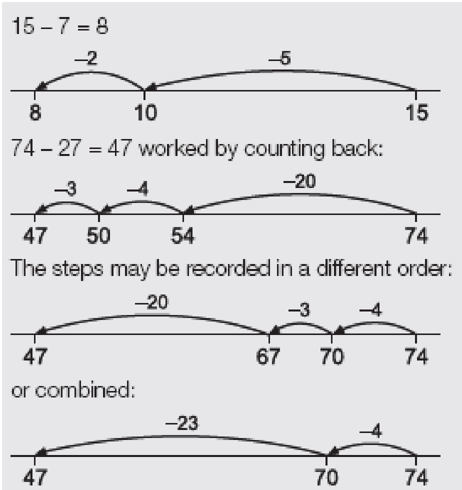
Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

13 – 5 = 8



The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.

Counting backwards

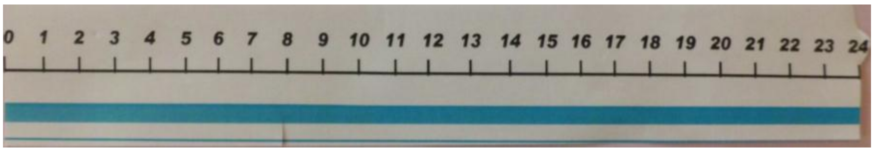


During the early stages of subtraction, counting backwards on a number line is the recommended strategy. However, it is expected that by the beginning of Key Stage 2 children have developed a concept of subtraction as finding the difference and should be encouraged to count forwards on a number line.

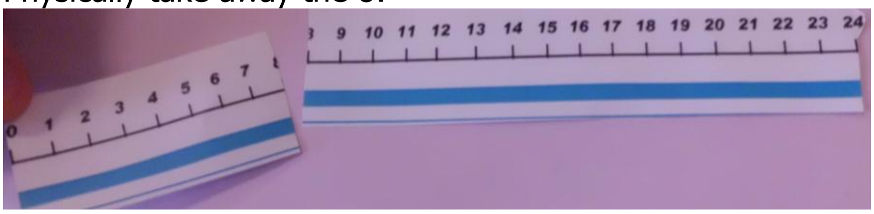
Finding the difference

Ensure that children understand why subtraction can be seen as finding the difference and why it makes sense to count up on a number line:

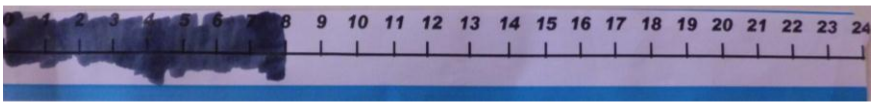
24 – 8



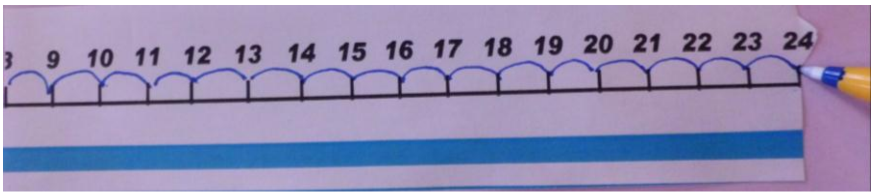
Physically take away the 8:

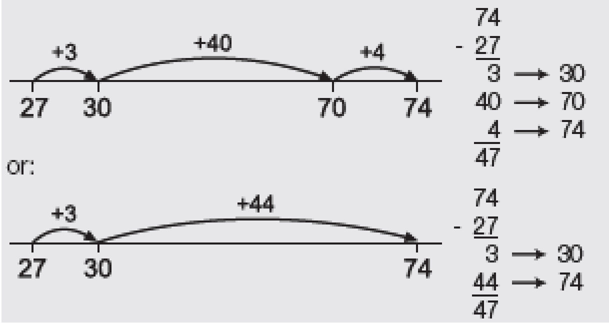


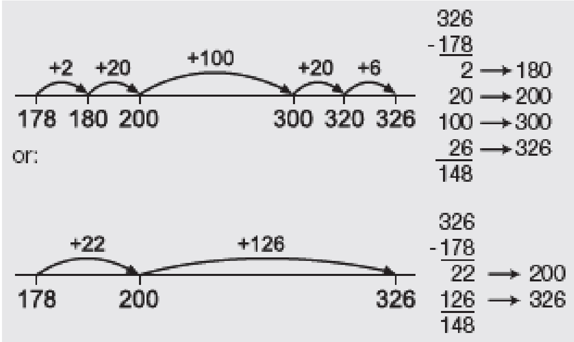
or



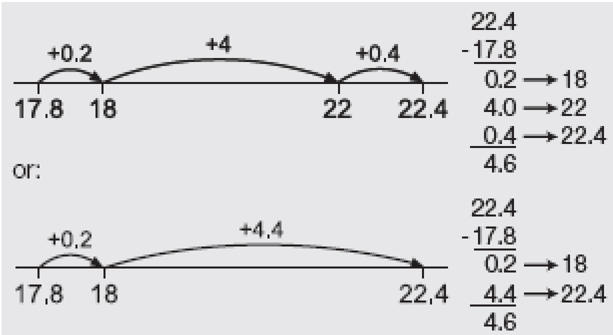
This is what is left. How can we work out how much this is worth? Count up from 8 to 24.







The method can be used with decimals where no more than three columns are required. However, it becomes less efficient when more than three columns are needed.



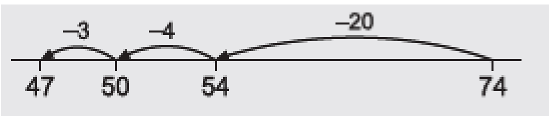
Stage 5: Partitioning

Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For 74 – 27 this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 7 in turn. Some children may need to partition the 74 into 70 + 4 or 60 + 14 to help them carry out the subtraction.

74 – 27 = 74 – 20 – 7 = 54 – 7 = 47

74 – 27 = 70 + 4 – 20 – 7 = 60 + 14 – 20 – 7 = 40 + 7

This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally.
The method of recording links to counting back on the number line, as shown below



Stage 6: Expanded layout, leading to column method

Please note: Continue to use HTU column headings until children are secure.

Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens.

Step 1:

Example: 74 – 27

$$\begin{array}{r} 70+4 \\ -\ 20+7 \\ \hline \end{array} \qquad \begin{array}{r} \overset{60}{\cancel{70}}+\overset{14}{\cancel{4}} \\ -\ 20+7 \\ \hline 40+7 \end{array}$$

Example: 741 – 367

$$\begin{array}{r} 700+40+1 \\ -\ 300+60+7 \\ \hline \end{array} \qquad \begin{array}{r} \overset{600}{\cancel{700}}+\overset{130}{\cancel{40}}+\overset{11}{\cancel{1}} \\ -\ 300+60+7 \\ \hline 300+70+4 \end{array}$$

Step 2:

$$\begin{array}{r} \overset{6}{\cancel{7}}\,\overset{14}{\cancel{4}} \\ -\ 2\ 7 \\ \hline 4\ 7 \end{array} \qquad \begin{array}{r} \overset{6}{\cancel{7}}\,\overset{13}{\cancel{4}}\,\overset{11}{\cancel{1}} \\ -\ 3\ 6\ 7 \\ \hline 3\ 7\ 4 \end{array}$$

The expanded method for three-digit numbers:

Example: 563 − 241, no adjustment or decomposition needed.

Expanded method	leading to
$\begin{array}{r} 500+60+3 \\ -\ 200+40+1 \\ \hline 300+20+2 \end{array}$	$\begin{array}{r} 563 \\ -\ 241 \\ \hline 322 \end{array}$

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'.

Example: 563 − 271, adjustment from the hundreds to the tens, or partitioning the hundreds:

Step 1:

$$\begin{array}{r} 500+60+3 \\ -\ 200+70+1 \\ \hline \end{array} \qquad \begin{array}{r} \overset{400}{\cancel{500}}+\overset{160}{\cancel{60}}+3 \\ -\ 200+70+1 \\ \hline 200+90+2 \end{array}$$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, and how 500 + 60 can be partitioned into 400 + 160. The subtraction of the tens becomes '160 minus 70'.

Step 2:

$$\begin{array}{r} \overset{4}{\cancel{5}}\,\overset{16}{\cancel{6}}\,3 \\ -\ 2\ 7\ 1 \\ \hline 2\ 9\ 2 \end{array}$$

Example: 563 − 278, adjustment from the hundreds to the tens and the tens to the ones:

<u>Step 1:</u>

$$\begin{array}{r} 500+60+3 \\ -\ \underline{200+70+8} \end{array} \qquad \begin{array}{rcrcr} & 400 & & 150 & & 13 \\ & \cancel{500} & & \cancel{50} & & 13 \\ & \cancel{500} & + & \cancel{60} & + & \cancel{3} \\ - & 200 & + & 70 & + & 8 \\ \hline & 200 & + & 80 & + & 5 \end{array}$$

Here both the tens and the ones digits to be subtracted are bigger than both the tens and the ones digits you are subtracting from. Discuss how 60 + 3 is partitioned into 50 + 13, and then how 500 + 50 can be partitioned into 400 + 150, and how this helps when subtracting.

<u>Step 2:</u>

$$\begin{array}{rccc} & 4 & 15 & 13 \\ & \cancel{5} & \cancel{6} & \cancel{3} \\ - & 2 & 7 & 8 \\ \hline & 2 & 8 & 5 \end{array}$$

Example: 503 – 278, dealing with zeros when adjusting:

Step 1:

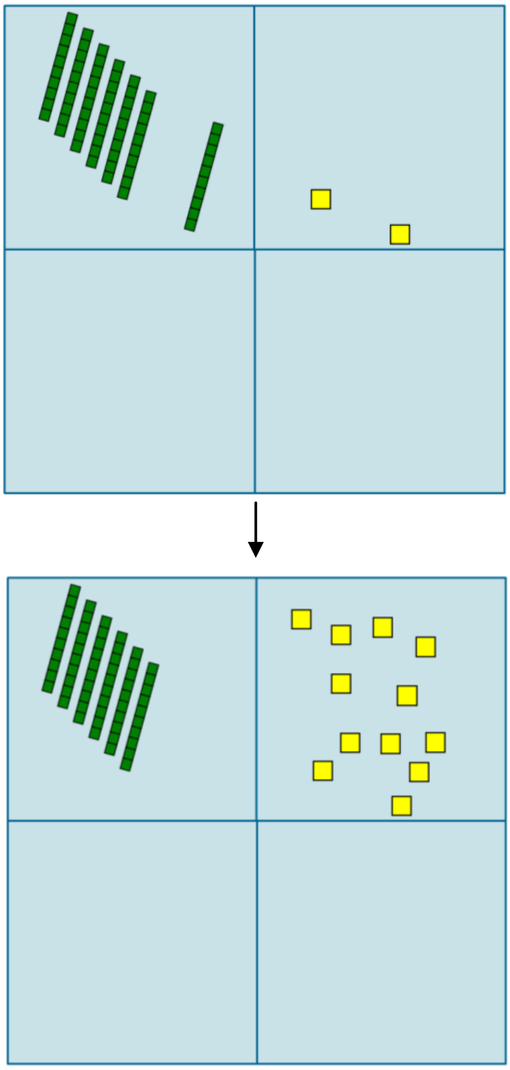
$$\begin{array}{r} 500 + \ 0 + 3 \\ -\ 200 + 70 + 8 \\ \hline \end{array} \qquad \begin{array}{rrrrr} \overset{400}{\cancel{400}} & & \overset{90}{\cancel{100}} & & \overset{13}{3} \\ \cancel{500} & + & \cancel{0} & + & \cancel{3} \\ -\ 200 & + & 70 & + & 8 \\ \hline 200 & + & 20 & + & 5 \end{array}$$

Here 0 acts as a place holder for the tens. The adjustment has to be done in two stages. First the 500 + 0 is partitioned into 400 + 100 and then the 100 + 3 is partitioned into 90 + 13.

Step 2:

$$\begin{array}{r} {\scriptstyle 9} \quad \\ {\scriptstyle 4\ \ 1\ \ 1} \\ \cancel{5}\cancel{0}3 \\ -278 \\ \hline 226 \\ \hline \end{array}$$

Model exchanging (adjusting) with Base 10 first:



$$\begin{array}{r} 72 \\ -\ 47 \\ \hline \\ \hline \end{array}$$

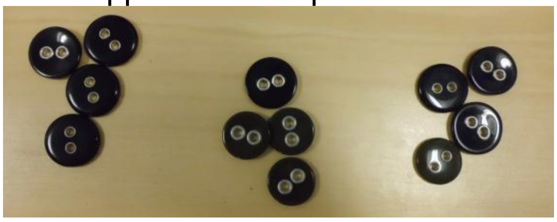
$$\begin{array}{r} \overset{6}{\cancel{7}}\,{}^{1}2 \\ -\ 47 \\ \hline \\ \hline \end{array}$$

Multiplication

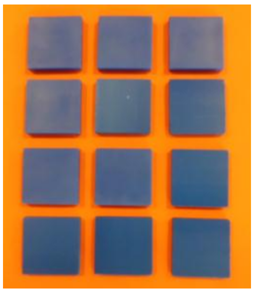
Stage 1: Concrete

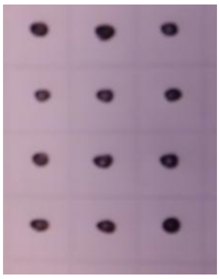
Children use real life objects and apparatus to explore the different models of multiplication.

'Lots' of or 'groups' of the same thing.

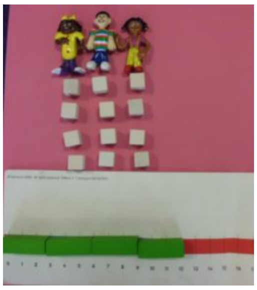


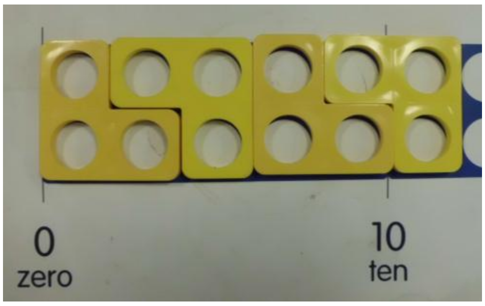
Arrays





Transition from concrete to numberline.





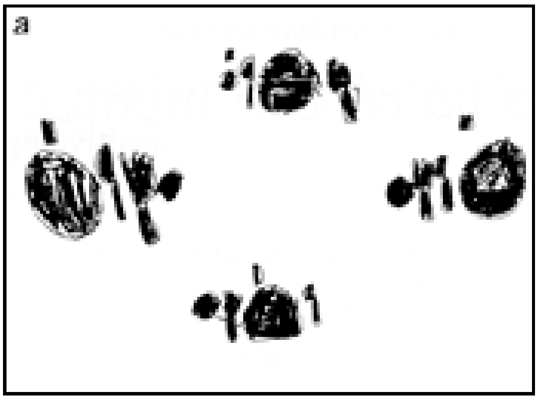
Multiplication as scaling.
The tree on the right is 2 times bigger than the tree on the left.

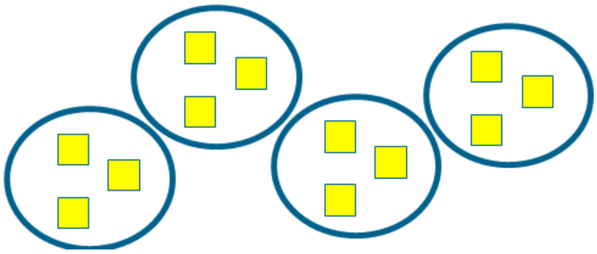
x 2

x 0.5

Stage 2: Pictures

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.





Singapore Bar Model:

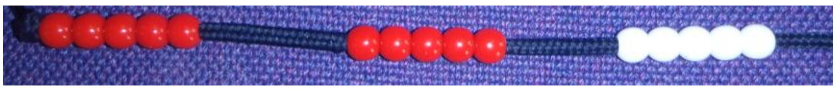
Emily has 7 stickers, Joe has six times as many stickers

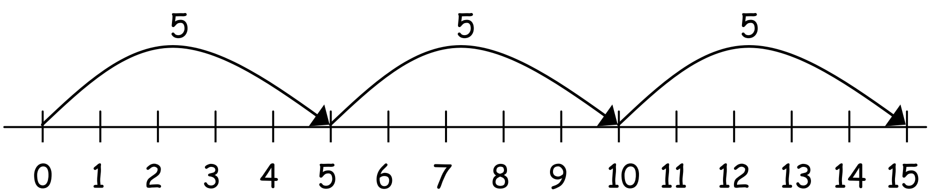
7	7	7	7	7	7
Joe's Stickers	Joe's Stickers	Joe's Stickers	Joe's Stickers	Joe's Stickers	Joe's Stickers
Emily's Stickers					
7					

Stage 3: The number line

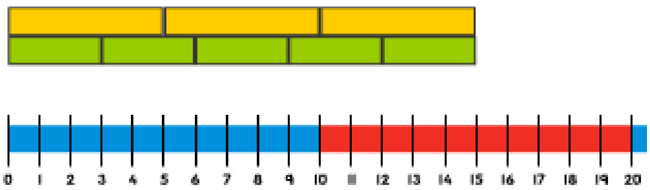
Repeated addition can be shown easily on a number line:

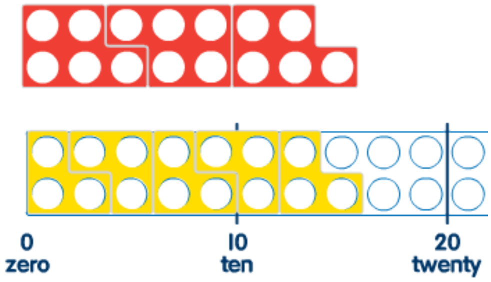
3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3

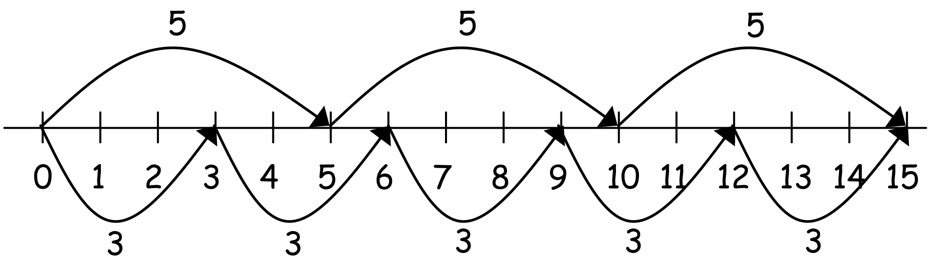




Children should know that 3 x 5 has the same answer as 5 x 3 (the commutative law of multiplication). This can also be shown on the number line.



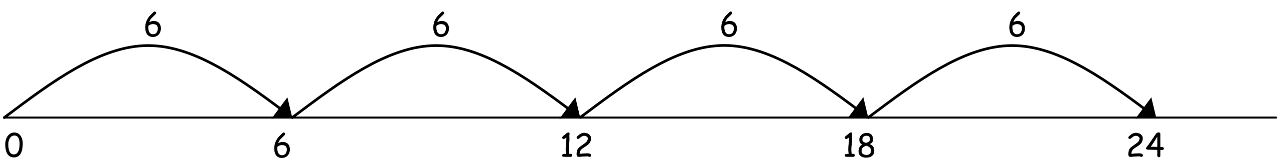




Stage 4: The empty number line

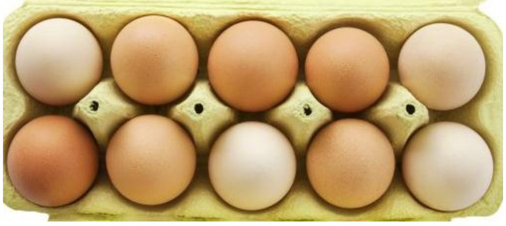
Children will continue to use repeated addition but will progress onto using a blank number line:

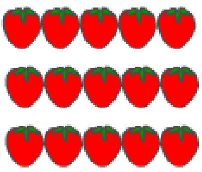
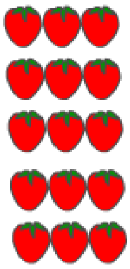
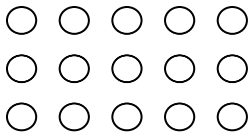
4 times 6 is 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4



Stage 5: Arrays

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method. Show children real-life arrays first:

5 x 3 = 15

3 x 5 = 15

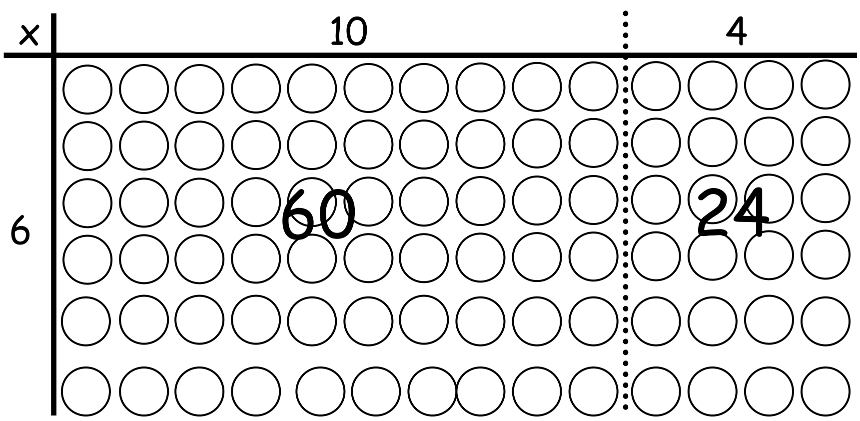
Stage 6: Mental multiplication using partitioning

Begin by recording mental multiplication using partitioning:

13 x 4 =
10 x 4 = 40
3 x 4   = 12
40 + 12 = 52

Stage 7: The grid method

Continue to use arrays, leading into the grid method for multiplication.
14 x 6 =

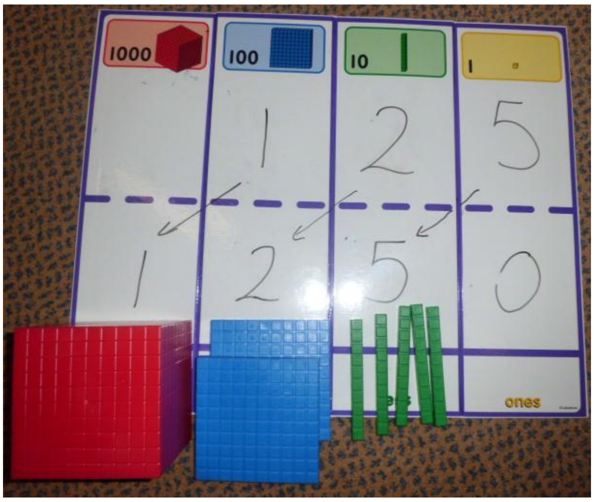


(6 x 10) + (6 x 4)

60   +   24

84

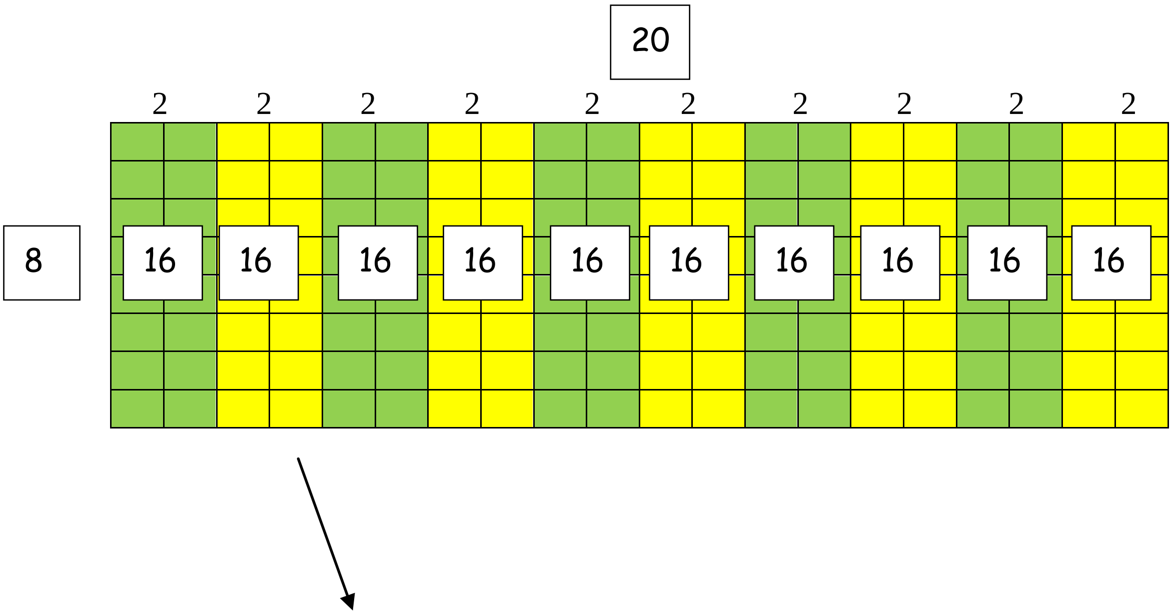
Before using the grid method, children should have a secure understanding of place value and be able to multiply confidently by multiples of 10. This should not be taught through how many zeros are placed on the end of a number; children should understand that as the digit moves to the left on a place value grid, its value becomes 10 times greater and the zero is a place holder.



TU x U

23 x 8 =

To complete the first box in the grid (20 x 8), children first need to understand that 20 x 8 is 10 times greater than 2 x 8. Use representations to help children to become secure with this concept:



x	20	3
8	160	24

$$
\begin{array}{r}
160 \\
+\ \ 24 \\
\hline
184 \\
\hline
\end{array}
$$

HTU x U
346 x 9 =

x	300	40	6
9	2700	360	54

```
  2700
+  360
+   54
  3114
  1 1
```

Using similar methods, children will be able to multiply decimals with one decimal place by a single digit number.

4.9 x 3 =

x	4	0.9
3	12	2.7

```
   12
+  2.7
  14.7
```

Stage 8: Expanded Long multiplication leading to long multiplication

The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method.

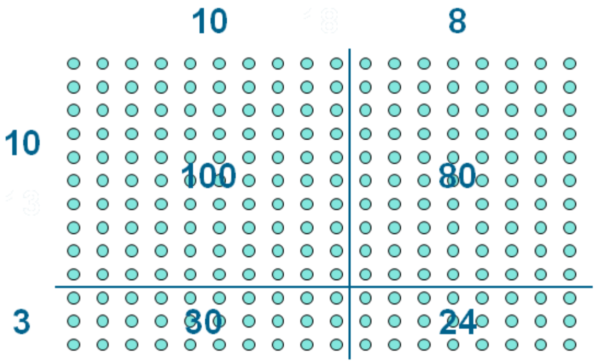
Children should describe what they do by referring to the actual values of the digits in the columns. For example, the second step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed.

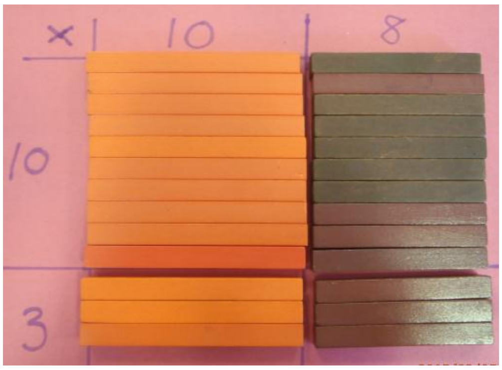
```
30 + 8                      38
 x   7                      x 7
    56  (8 x 7 = 56)         56
   210  (30 x 7 = 210)      210
   266                      266
```

Stage 9: Two-digit by two-digit calculations

Again, make sure children have related the grid method for two-digit by two-digit calculations to the array representation before moving onto the more formal method.

18 x 3 =





x	10	8
10	100	80
3	30	24

100
+ 80
+ 30
+ 24
234
1

72 x 38 =

x	70	2
30	2100	60
8	560	16

2100
+ 560
+ 60
+ 16
2736
1

Reduce the recording, but always showing the links to the grid method. Order of steps, as below:

```
   56
x  27
   42    (7 x 6 = 42)
  350    (7 x 50 = 350)
  120    (20 x 6 = 120)
 1000    (20 x 50 = 1000)
 1512
  1
```

Stage 10: Three-digit by two-digit calculations

372 x 24 =

x	300	70	2
20	6000	1400	40
4	1200	280	8

```
   6000
+  1400
+  1200
+   280
+    40
+     8
   8928
   1
```

Reduce the recording, but always showing the links to the grid method.

```
 286
x 29
  54    (9 x 6 = 54)
 720    (9 x 80 = 720)
1800    (9 x 200 = 1800)
 120    (20 x 6 = 120)
1600    (20 x 80 = 1600)
4000    (20 x 200 = 4000)
8294
1
```

Using this method, children will be able to multiply decimals.

```
 286
x  2.9
    5.4    (0.9 x 6 = 5.4)
   72.0    (0.9 x 80 = 72.0)
  180.0    (0.9 x 200 = 180.0)
   12.0    (2 x 6 = 12.0)
  160.0    (2 x 80 = 160.0)
  400.0    (2 x 200 = 400.0)
  829.4
  1
```

Stage 11: Long multiplication

The recording is reduced further, with carry digits recorded below the line.

```
   38
x   7
  266
   5
```

Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU.

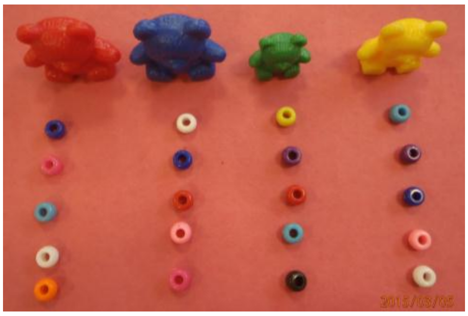
```
  286
x  29
 2574    (9 x 286 = 2574)
  7 5
 5720    (20 x 286 = 5720)
 1 1
 8294
 1
```

Division

Stage 1: Concrete

Children use real life objects and apparatus to explore the different models of division.

Sharing:



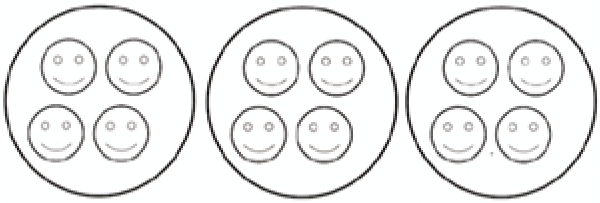
Grouping:

20 ÷ 4 = 5
(Share 20 counters equally between 4)
How many each?

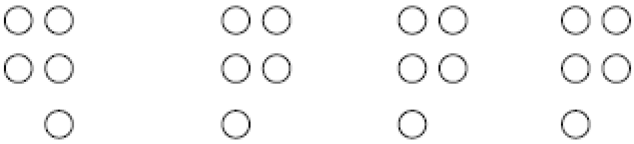
20 ÷ 4 = 5
(Put 20 counters into groups of 4)
How many groups of 4 in 20?

Stage 2: Pictures

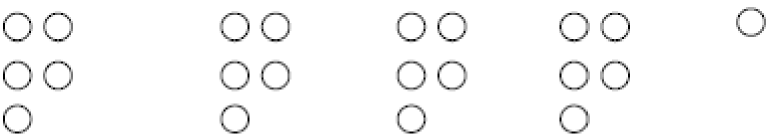
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.
12 ÷ 4 = 3



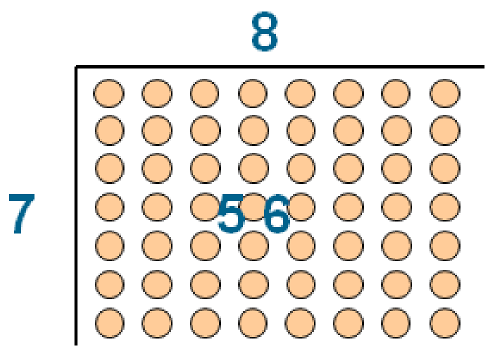
20 ÷ 5 = 4



21 ÷ 5 = 4 r 1



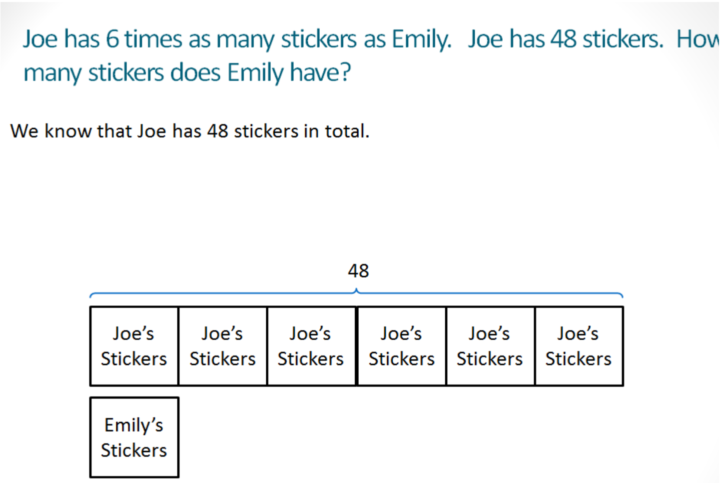
The array is an image for division too



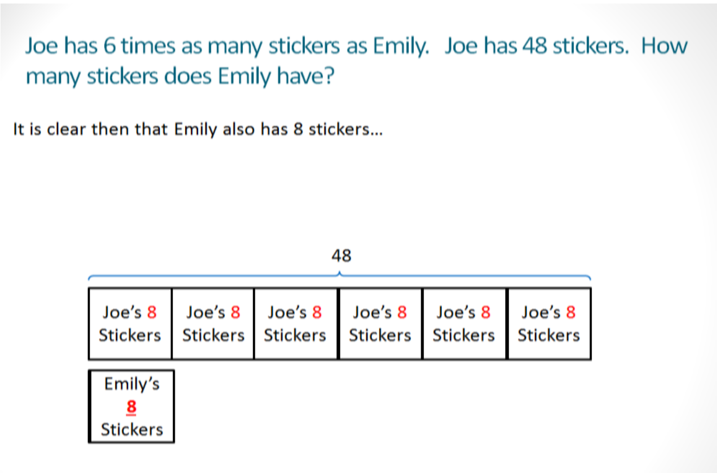
$$7\overline{)56}^{\;8}$$

This representation can be used to show why division calculations are sometimes laid out like this. We are looking for the missing side of the array.

Use the Singapore Bar as a representation to solve division problems. This model can be used at ALL STAGES as a tool for children to work out what calculation they need to carry out:



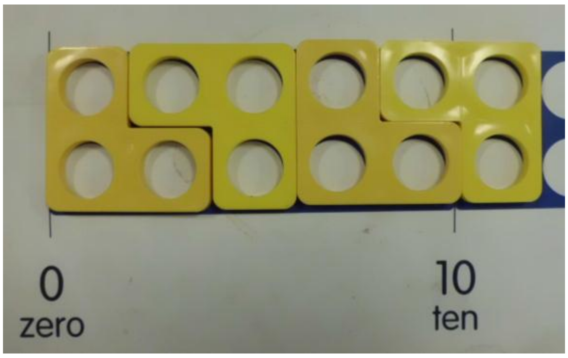
The model shows us that we must calculate 48 ÷ 6

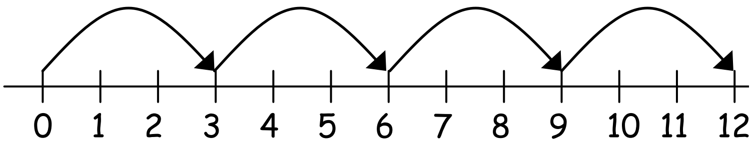


48 ÷ 6 = 8

Stage 3: The number line

12 ÷ 3 = 4
Count up on the number line to see how many 3s are in 12.
Use different representations of the numberline:

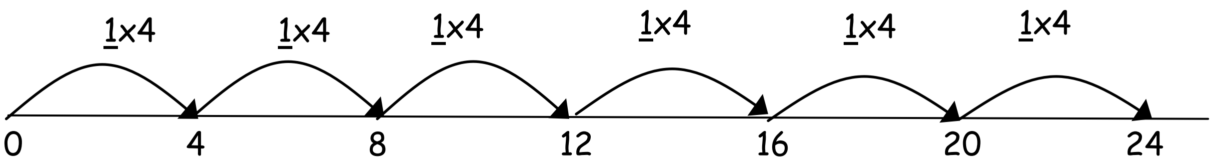




Stage 4: The empty number line

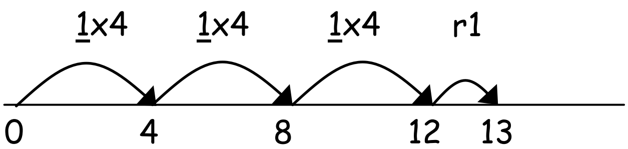
Children will progress onto using a blank number line:

24 ÷ 4 = 6



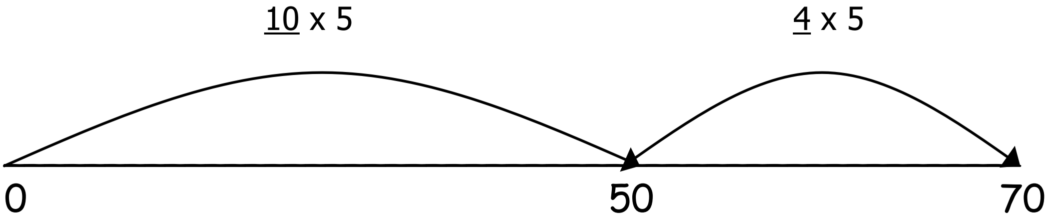
Children should also move onto calculations involving remainders.

13 ÷ 4 = 3 r 1



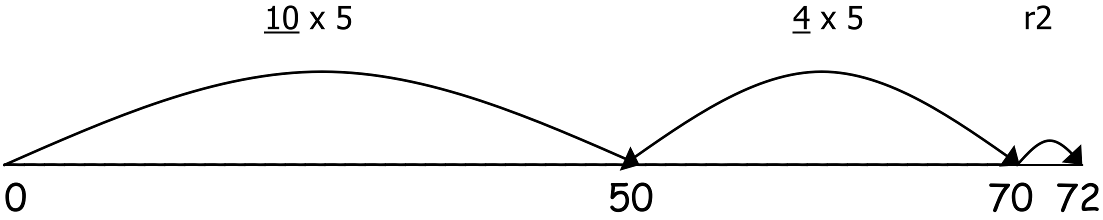
$70 \div 5 = 14$

Moving onto chunking on a numberline. Relate to the inverse of multiplication. Ask, how many 5s in 70? 10 lots of 5, 4 lots of 5. How many lots of 5 altogether? 14.



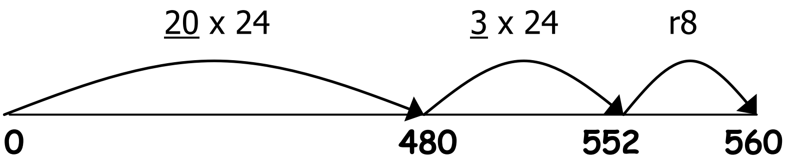
Division with remainders:

$72 \div 5 = 14 \text{ r } 2$



HTU ÷ TU

$560 \div 24 = 23 \text{ r } 8$



Tip: Children can jot down coin multiplication facts to give them a bank of facts to use on their number line. Coin Multiplication takes a given number and multiplies it by 1, 2, 5, 10, 20, 50 and 100.	**1 x 24 = 24** **2 x 24 = 48** **5 x 24 = 120** **10 x 24 = 240** **20 x 24 = 480** **50 x 24 = 1200** **100 x 24 = 2400**

Children who are confident with this method may move onto recording vertically, in the style of long division:

```
24 ) 560
   - 480     24 x 20
      80
   -  72     24 x 3
       8
Answer: 23 R 8
```

The answer can be recorded above the line to mirror short division.

```
       23
24 )  560
    - 480
       80
     - 72
        8
Answer: 23 R 8
```

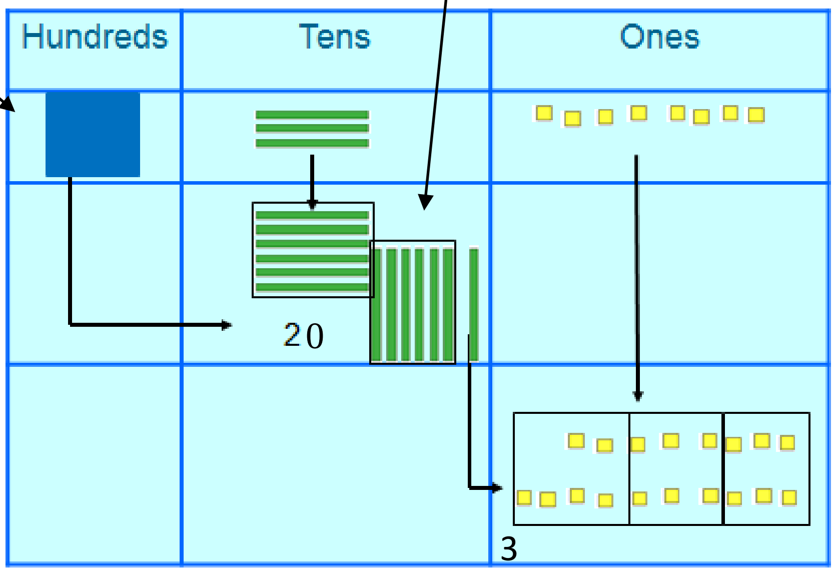
Stage 5: Short division

'Short' division should be introduced as a more compact recording of division when children are confident with multiplication and division facts and their understanding of partitioning and place value is sound. First, they should be able to confidently divide by chunking on a number line or vertically.

Use Base 10 representations to show what is happening: $6\overline{)138}$

1. Although we know that we can fit sixes into 100, this 100 square cannot be physically broken into groups of 6 so we exchange it for 10 tens.

2. We now have 13 tens. We can break them into 2 groups of 6 tens with 1 ten left over, which we exchange for ones.
The 2 groups of 6 tens represent 20 lots of 6. We write the 2 in the tens column above the line in our calculation.



$$\begin{array}{r} 2\ \ 3 \\ 6\overline{)\,\cancel{1}^{0}\ {}^{1}3\ {}^{1}8} \end{array}$$

3. We now have 18 ones. We can group these into three lots of six. So we write 3 in the units column above the line in our calculation.
Altogether there are 23 sixes.

$$\begin{array}{r} 2\ 7 \\ 3\overline{)\,8^{2}1} \end{array} \qquad \begin{array}{r} 0\ 2\ 3 \\ 24\overline{)\,5^{5}5^{7}2} \end{array} \qquad \begin{array}{r} 0\ 2\ 3\ r\ 8 \\ 24\overline{)\,5^{5}6^{8}0} \end{array}$$

Children can then progress to finding an answer with a decimal:

$$\begin{array}{r} 2\ \ 2.\ 5 \\ 6\overline{)\,1^{1}\,3^{1}\,5.^{3}\,0} \end{array}$$